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THE MATHEMATICS TEACHER

A MAGAZINE DEVOTED TO THE INTERESTS OF TEACHERS OF MATHEMATICS

Volume IV December, 1911 Number 4

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THE MATHEMATICS TEACHER

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INTERESTS OF TEACHERS OF MATHEMATICS

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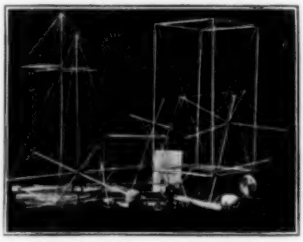

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THE MATHEMATICS TEACHER

EDITED BY
W. H. METZLER

ASSOCIATED WITH
EUGENE R. SMITH MAURICE J. BABB

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EDITORIAL.

Proofs of the Pythagorean Theorem

Mr. Arthur R. Colburn, an attorney of Washington, D. C., has taken as his recreation the study of the Pythagorean theorem and its proofs. He started with a knowledge of but two proofs, those of Pythagoras and Euclid, and has rediscovered a good share of the well-known proofs, and a number of others, some of which are little known, if not entirely new.

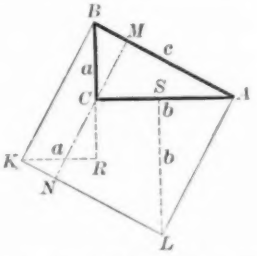
Mr. Colburn claims to have seventy-three proofs, but this number is excessive because some of his proofs are duplicates, in whole or in part, of one or more of the others, or are such that if they are reduced to the essential minimum they become duplicates. Despite this fact, his work might well act as a spur to many teachers who seem to consider geometry as a subject allowing little chance for investigation, owing to its centuries of study by master minds. Even our best teachers might well profit by spending some time in such examination of theorems and the methods of discovering their proofs, for after all, the methods of attack, and the development of original ideas are the great aim of geometrical thought, and unless one is himself in this habit of mind, it is doubtful whether he can give it to others.

One of the most interesting parts of Mr. Colburn's work is a theorem that he states and proves, and by which he finds a number of simple proofs for the Pythagorean theorem. The

theorem is not commonly known, and may be new in this form, although such a claim is hard to establish. It is as follows:

If from one end of a side of a parallelogram a line is drawn to the opposite side (or its extension), and from the other end of that side a perpendicular is drawn to the new line (or its extension), the parallelogram is equivalent to the rectangle of those lines.

The proof is simple, using the fact that parallelograms having equal bases and equal altitudes are equivalent, applied twice, and its application to the Pythagorean theorem is instant, varied, and interesting. The following illustration will show its use:



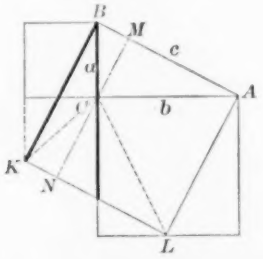
The right triangle ABC is given; the square $ABKL$ is constructed on AB, and MN is drawn perpendicular to AB through C. Then BC (or a), and CA (or b), are lines from a vertex of a parallelogram to the opposite side, so by Colburn's theorem $MBKN$ and $AMNL$ are respectively equivalent to the rectangles of these lines by the perpendiculars to them from K and from L. But these perpendiculars, KR and LS, are easily shown by the triangles BRK and ASL to equal a and b respectively. Therefore $MBKN$ is equivalent to the square on a, and $AMNL$ to the square on b, and the theorem is proved.

Whether this theorem has any other applications that are of value in simplifying the subject is doubtful, although it can be applied in other proofs dealing with squares; for example, it will prove, by equivalence instead of proportion, that the square on a leg of a right triangle is equivalent to the rectangle of the hypotenuse by the projection of the leg on it. The theorem is itself a special case of a more general theorem, which does not seem, after a brief examination, to have any particular value.

In dealing with proofs of the Pythagorean theorem it is difficult to decide how small a variation constitutes a difference in

proof. There can be no doubt that some of the best known proofs are variations of each other, rather than distinct proofs, and it would be an interesting investigation to examine fully all possible methods of proof, grouping them by the principles used. For example, any one of the ordinary equivalence methods that uses the three squares can be proved with six different figures, the method of proof remaining unchanged, as follows:

1. With the square on the hypotenuse outside the triangle, and
   (*a*) the squares on both legs outside,
   (*b*) the square on one leg inside, the other outside,
   (*c*) the squares on both legs inside.
2. With the square on the hypotenuse inside the triangle, and
   (*a*) the squares on both legs outside,
   (*b*) the square on one leg inside, the other outside,
   (*c*) the squares on both legs inside.



For example using 2, (*a*) the Euclidean proof would become, briefly, $BCK = \frac{1}{2}$ square on *a*, and equals $\frac{1}{2}$ *MBKN*, $ACL = \frac{1}{2}$ square on *b*, and equals $\frac{1}{2}$ *AMNL*, therefore *ABKL* (or square on *c*) is equivalent to square on *a* plus square on *b*. In this figure the two triangles that are usually proved congruent have become identical.

Furthermore, the proofs that use the square on the sum of the legs, or any part of that figure, could be grouped, and all possibilities, both of arrangement of parts within the figure, and of omission of various parts not essential to a proof could be listed. If each principle of plane geometry that could apply were thus examined, with all such possibilities for each being considered, the ground would be completely covered. Among the unusual ways of proving this theorem, that, though of no practical importance, may be of interest, might be mentioned the use

of the following propositions, all of which can be proved without its use.

The sum of the squares on two sides of a triangle equals twice the square on half the third side plus twice the square on the median to that side.

The sum of the squares on the sides of a parallelogram equals the sum of the squares on the diagonals.

The sum of the products of the opposite sides of an inscribed quadrilateral equals the product of its diagonals.

The product of two sides of an inscribed triangle equals the product of the altitude to the third side by the diameter.

The product of two sides of a triangle equals the square of the bisector of the included angle plus the product of the sects of the third side.

The wonderful possibilities of this theorem, when examined in a logical way, ought to impress us with the almost infinite richness of geometry; the possibilities of the subject multiply almost to infinity, and the teacher who cannot find material to interest himself and his class in investigations of some such nature is blind to his opportunities.

The Mathematics Section of the Bibliography of Science Teaching

This bibliography was compiled by a committee of the American Federation of Teachers of the Mathematical and the Natural Sciences, and was published by the United States Bureau of Education. While the report lists some excellent works on the elementary and secondary mathematics, it seems a pity that the opportunity was not taken to furnish the teacher with a broader knowledge of recent important committee reports, articles bearing on both sides of the various movements that have aroused discussion, the periodicals that are publishing mathematical subject matter, and, in fact, all that the live teacher would find of interest.

The point most likely to be criticized is that almost all of the works published in the United States that are listed are written by men centered around two of our important universities, and while these works are probably all of sufficient importance to justify their being included, others that many would consider of at least equal importance are excluded. It does not seem

reasonable that there can be such a dearth of material throughout all the rest of our educational system.

Furthermore, the reports of certain associations are included, while other reports that have had at least as strong an influence do not appear. Reports from the American Mathematical Association, and from the Central Association are on the list, as they should be, but why are those of the New England Association committees on arithmetic and geometry lacking, and why should the reports of the Middle States and Maryland Association committees on algebra and on marking examination papers be ignored? Why are teachers not given advance notice of the Federation's own committee on geometry as well as of the work of the International Commission on the Teaching of Mathematics? Again, if the bulletins of one state are included, why should those of other states fail to obtain notice? Many teachers outside of New York State would be interested in the definitions laid down by the Board of Regents, others would be glad to examine the content required for mathematics courses in New Jersey high schools, while all would be likely to be interested in knowing exactly how many states have issued definite statements on mathematics, and how the various states compare in their recommendations. Some of the schools outside of its usual territory may not be familiar with the definitions of the College Entrance Examination Board, and so on. Is it not to list just such publications as these, to call attention to many of the interesting articles appearing in the educational magazines, and, in short, to do for the teacher what he would find it hard to do for himself that this committee was appointed? In looking over the other sections of the bibliography, it appears—although a mathematical magazine is hardly in a position to speak with authority on biology, nature study, and other natural sciences—that considerable work of this kind has been done. It seems, however, that the material would have been of much more value to the one using it if it had been indexed by subjects, and perhaps by source, rather than by authors. If room could be found for that also, so much the better.

To sum up, it seems that the report is very incomplete, and that it falls far short of being of such value to the elementary and secondary teacher as its intended importance would seem to justify. S.

STUDY SUPERVISION: ITS NEED IN THE MATHEMATICS OF THE ELEMENTARY AND SECONDARY SCHOOLS.

By A. M. Curtis.

The remarks of the last speaker in emphasizing the importance given by Harvard to ability of the student to accomplish in limited time definite tasks are most timely in view of the point I wish to bring to your attention under the title on your program.

The school work of the seventh and succeeding years has been so planned in many schools that study under the restraints of the school-room has been given over to the pupil's own free methods on his own time.

The results of this unorganized work are brought to the teacher at the recitation period and are inspected by her, if in written form, or tested by question in oral exercise. Such lessons are prepared by the pupils on any sort of paper and by any methods they happen to devise, whether good or bad. The main object being to copy up in immaculate form a finished product acceptable, first, to the teacher's zealous eye for form and, second, for a "passing" number of correct "answers."

Such work is pernicious. It leaves the vital element in the training, which mathematics ought and, if properly taught, does give, entirely to a haphazard, makeshift chance.

Such copied work gives little insight into a pupil's real ability in work habit and power. It never can tell the teacher how many trials were necessary in solving a problem; how much time was wasted by aimless manoeuvring; nor whether a bungling, crude, roundabout method of computation was employed where a brief reflection and mental calculation would and ought to find application. In fact this copied home work reveals but little. The work the teacher should see is to be found in the home waste basket, on rough paper, perhaps slovenly in appearance, and wrongly computed. This point is so important I illus-

trate it by the following comparisons:

Problem: At $3½ per head, what will 17 sheep cost?

Papers (copied work) give, perhaps uniformly the following statement:

$17 \times \$3½ = \$59½$, cost, Ans.

If, however, the teacher were to see the original work, differences would appear somewhat as follows:

Pupil A—$17 \times 7/2 = 119/2 = 59½$.

Pupil B—3.5
17
245
35
59.5

Pupil C—17
3½
8½
51
59½

Pupil D—$17 \times 3½ = 51 + 8½ = 59½$.

Which of the pupils has the best practical solution? The others need but a word and they may be brought to see the value of mental power in saving work.

Problem: At 2½c each, how many yards can I buy with $6.50?

Different solutions follow:

A solves this way, producing error thereby:

.025) 6.5 (26.
50
150
150

B solves this way:

260.
⊙025.) 6⊙500.
5 0
1 50
1 50

C thinks: $1 buys 40 yd.
($6 buys 240 yd.
Writes.... ($.50 buys 20 yd.
($6.50 buys 260 yd.

C soon learns to write nothing but the result, all work being done mentally.

All the *copied* work for these pupils is the same.

A $\$6.50 \div \$.02\frac{1}{2} = 26$, therefore 26 yd. Cause of error not apparent.

B $\$6.50 \div \$.02\frac{1}{2} = 260$, therefore 260 yd.

C $\$6.50 \div \$.02\frac{1}{2} = 260$, therefore 260 yd.

Is not C's work, as *originally* thought out, in good form? Compare B's original form with A's.

I have dealt for sixteen years with high school graduates who attempt to study arithmetic methods. Of all those I have met not over twenty-five per cent. show any knowledge or habit of thought better than A or B in the illustrations above. Many can not divide by 8 in short division without placing the carrying figure on the paper.

This very experience with pupils in our school made me realize that nobody had *ever watched the working methods and habits* of these pupils.

How accomplish this watchful, helpful work in our schools?

There is but one answer. Supervise the study. And I suggest that a recitation period used frequently as a study period where the pupil works under careful restriction with definite aim as to first-time accuracy, first-time neatness, and speed, will accomplish much better results than hours of harum scarum work at home.

As to the details of such a plan, I have found the following helpful. When the study period arrives, clear the desks of all paper but one sheet and the problems. Have the work done *in ink* (no pencils in service). It is evident that this simple means forces thought before action; for a wrong attack is shown by the non-erasable ink. Neatness and long process methods are not compatible. Nor is the use of pencil in extended work conducive to the rapidity necessary for finishing the task within the allotted time.

This plan permits and suggests many a valuable exercise; for example, thinking a problem through "in the rough" to arrive at an approximate result. The teacher can give so many minutes for these first results; have them written in *ink* on the paper

for the lesson, then have the remaining time given to *accurate* solution, which proves the "guess" result either close or "wild."

Such supervision is not for arithmetic alone. Nothing will put better training into algebra than this same first-time accuracy and neatness. Carelessness in sign changes, in exponent arrangement, and in the habit of verification will all be revealed to the one whose business it is to insist on the opposite traits; namely the teacher.

In geometry what more necessary exercise can be used than careful study on the following plan?

With the book-demonstrated proposition, have the pupils cover all but the theorem. Read the theorem, give hypothesis, conclusion, and construct his figure in harmony with his analysis. Now remove the cover from the figure and compare. Note the lines of construction in the text figure and speculate on their significance. Suggest lines of proof. Next study the author's proof.

Such careful teaching and habit of study give results worth while and independent pupils, book or no book. But supervision is the essential of the scheme.

To summarize this talk:

Supervision of study on the simple lines described will gain for the pupil a habit (1) of thought before action, (2) of neatness (and consequently conciseness) in form, (3) of accuracy under stress of time limit. In short, the study of mathematics *will stand for increase in power*, such as our good Harvard friend mentioned but now.

STATE NORMAL SCHOOL,
ONEONTA, N. Y.

THE WAY TO BEGIN SOLID GEOMETRY.

By Howard F. Hart.

The methods in solid geometry which I am now using in our high school have proved to be such an advance over former methods that I have called this paper, which describes those methods, by the presumptuous title above.

It has always seemed to me that the aims in solid geometry should be to continue those of plane geometry together with the visualization of the figures as they in reality exist in three dimensional space. I believe that readymade solutions for definitions, propositions, and exercises should not be given in advance to the students but should be the summaries of class discussions on those points. In the beginning of things it must have been that the figures were studied first and the theorems derived from them. Therefore the figures should precede. Furthermore, it should be directly or tacitly assumed that the figures studied exist. This is as logical and better than the present practice in which the existence of certain things is discussed at great length (*e. g.*, a perpendicular to a plane) while others just as vital and necessary (*e. g.*, the intersection of two planes or the intersection of a spherical surface by a plane in a closed line) are assumed. However, such propositions on existence as *e. g.*, the common perpendicular between two non-coplanar lines and the number of ways in which regular plane polygons can be used to form the faces of polyhedral angles should be discussed and demonstrated as they are different in kind.

So after a review of the logical relations of the converse, negative, and negative-converse of a proposition; the locus idea and what has to be proved in order to prove a locus; the loci of plane geometry; the definitions of solid, surface, plane surface (two definitions, for both are necessary) and straight line; and a discussion of what classes of plane geometry propositions are applicable to figures lying in more than one plane; the first

figure "two straight lines in space" is discussed. The class readily finds the three possibilities.

Of course in this early work, and constantly throughout, we have to be alert to use whatever material is at hand to help the student build up his space concepts. The class room is a never-failing source of help. The lines of the ceiling, side walls, and floor give all that is needed in many configurations. Strings from the gas jet to a desk are very helpful. Whatever models are used are such that they can be built up before the class. In this connection I find Hanstein's goniostat (Chicago Math. Supply House) to be very helpful and can commend it.

To return to our first figure: Two straight lines in space. As given above, there are three possibilities: (1) and (2) in the same plane, hence parallel or intersecting; (3) not in the same plane. I have found it very easy to deduce the fact that each of the first two possibilities determines a plane while the third makes it impossible for the lines to be either parallel or intersecting.

Then we study "Two Planes in Space." They may or may not have common points. The students naturally apply the right name to the second possibility and thus parallel planes have become in the right way (Dewey says in "How We Think" that anything learned without the use of thinking, *i. e.*, induction, is not educative) a part of the student's notions and the definition is a mere incident. A deeper questioning will develop the fact that the intersection of two planes must be defined as the locus of their common points. At this point I tell the class that since they seem to be unable to think of two planes as having only one common point, we will assume that they have two such points. From that it is but a step to see that all points on the line determined by these two points are common, that any common point is on the line, and that the line is therefore by definition the intersection of the planes. This is one new locus. Also we see what hereafter we must know about a line to call it the intersection of two planes.

For the third figure we consider the relative positions of a "Line and a Plane." The line may lie in the plane, be parallel to it, or intersect it. We find that apart from the definition (a plane is such that a straight line joining, etc.) a line lies in a plane if it is parallel to a line in the plane and has one

point in the plane but is parallel to the plane if parallel to the line and has no point in the plane. I find that the students in considering the third possibility without any questioning naturally subdivide it and name the cases. Then we hunt for the conditions which a line must satisfy to be perpendicular to a plane. I find that at this time the students need to have this question asked of them, "What would be the condition of a line which was known to be *not* perpendicular to one line in the plane through its foot." They see that the line *could not be* perpendicular to the plane ("It must slant" as one boy put it), and hence get the definition. Set to work to see how many lines it must be perpendicular to be perpendicular to all they soon find the answer. I reserve the discussion of the line oblique to a plane until after we have investigated the projection of a line on a plane.

The fourth configuration is that of "Three planes in space." The three planes may obviously intersect in three lines, in two lines, in one line, or not at all. If they intersect in three lines the lines the lines of intersection may either be concurrent or parallel as two of them are concurrent or parallel. The second of these alternatives is very important in later study of parallels. If the planes intersect in two lines then two of the planes do not intersect and are therefore parallel and the lines are parallel by definition. The third possibility gives us the arrangement called "Pencil of Planes," a useful idea. In the last the planes are all parallel to each other.

For the fifth subject we study the "Lines perpendicular to one line at the same point." Determining our planes carefully we find that (1) the given line and any one of the other lines are coplanar, (2) any three of the latter are coplanar, and that therefore they are all coplanar. The converse has already been studied in a line perpendicular to a plane; hence we can take this as a new locus at once.

These first five studies in the relative positions of lines and planes in space are doubtless enough of the beginning to indicate the aims and methods of the plan. It will be seen that we begin at the *beginning;* that thinking has been a maximum, telling a minimum; that the study of the figures produces theorems whose proof is but the orderly arrangement of the discussion.

I assume that it will be understood that many a class period is given over to formal recitations in which the orderliness of the work is carefully criticized, that many exercises are given in further relations of the figures and later, particularly, in mensuration. Finally let me say that success in solid geometry depends upon insight into the figures; that I know of no way to give a boy such an insight except to develop it.

And it can be developed.

HIGH SCHOOL,
MONTCLAIR, N. J.

VICE.

The shadow of idleness.—Tennyson.

The daughter of pleasure.—Bushnell.

The tax-gatherer of the community.—Anon.

The greatest of all Jacobins; the arch-leveller.—Hare.

A burdensome fellow-traveller, a chargeable table-companion, and a troublesome bed-fellow.—Plutarch.

Such a self-sufficient worker of infelicity, that it has no need either of instruments or servants.—Plutarch.

WHAT IN HIGH SCHOOL MATHEMATICS IS OF MOST IMPORTANCE AS A PREPARATION FOR ANALYTIC GEOMETRY AND THE CALCULUS IN COLLEGE.

By Arthur Sullivan Gale.

One of the educational problems of the day, demanding solution with increasing insistence, is the question of the inter-relations of diverse college entrance examinations and the complex curriculum of the modern high school. This general problem is immediately suggested when a college instructor proposes to indicate the portions of the mathematical subjects commonly taught in the high school which are of most importance for the study of analytic geometry and the calculus. For such an indication implies that the colleges would like to have these portions of the elementary branches emphasized in the high school, and I can almost hear some high school instructor, on looking over the program, say to himself, "An attempt of the college to impose further unreasonable demands on the high school." Far be it from me to consider the topic proposed in any such spirit. Rather, let the discussion be conducted on the lines, fostered by this and other similar associations, along which progressive high school and college instructors are working harmoniously to secure in the high school a curriculum adapted to the needs of the large body of students who do not go to college and to the adequate preparation of the comparatively few who do.

With regard to the general problem mentioned above I shall maintain a discreet silence, my only reason for referring to it at all being to express the hope and belief that a frank discussion of the question under consideration, and other similar questions, will ultimately be of use in the solution of the general problem. That this discussion may be of such use is clear. For if the topics taught in the high school of major importance to the pursuit of analytics and calculus have been listed, the remaining topics are of relatively little or perhaps of no impor-

tance in that connection. If in the latter list there appear topics which are of little significance for the student not preparing for college, and if these topics are commonly included in college entrance requirements, I believe that the high schools are justified in asking that such topics be dropped from the entrance requirements. To this some college instructors may object that while certain topics are not used in analytics and calculus, nevertheless they are important in the development of an effective mathematical technique. But I think that it is entirely possible that this essential technique may be developed as well by other topics which may be of greater importance to the student whose formal education is completed by the high school course.

If anything further is needed to indicate that I do not approach this discussion with the thought that it affords an opportunity to criticize instruction in the secondary schools, let me say that I believe that many secondary schools, are teaching mathematics very effectively. As evidence, if any be needed, I would offer the fact that each year I find that one of the two best students in the sophomore course in analytics and calculus is a Freshman who has anticipated the mathematics of the Freshman year by courses in the high school. Let me also point out in this connection that in so far as the discussion relates to solid geometry, trigonometry and advanced algebra, it affects colleges, but not technical schools, quite as much as the secondary schools.

Although it would be possible and profitable to spend the time allotted me in answering the proposed question in general terms, I believe that it is more desirable to consider questions of detail. In what follows the term "important" refers, of course, to the preparation for analytics and calculus. Thus the graphical representation of complex numbers is of no importance in this connection but it is fundamental from other viewpoints.

Let us begin with algebra. It is almost superfluous to say that the utmost importance attaches to the four fundamental operations of addition, subtraction, multiplication and division. If these operations be applied to any set of numbers, whether they be denoted by a, b, c, and x, y, z or by more complicated symbols such as $\sqrt{ax+b}$, $(ax^2+bx+c)^{-3/2}$, 2^x, $\log x$, $\sin 2a$,

and arc sin $(a+b)$ being immaterial, we generally obtain fractions, and the problem of the simplification of complex fractions arises. An essential for the study of calculus is the ability to simplify fractions which may be denoted, when compared to the complex fractions in our elementary texts, as hyper-complex. I am restrained from giving way to the feelings aroused by the errors of the poorer students out of the fulness of my own experience. Even after I had been in college some time I approached a fraction with a respectful reluctance. After finishing the first course in calculus I turned to the algebra I had studied to look at the complex fractions which I remembered with something akin to horror, but I was unable to find them. After convincing myself that I had in hand the book I had studied and that there were no pages missing, I was amazed to discover that nearly all the complex fractions in the book could be simplified by inspection. Instead, therefore, of urging a drill in the simplification of more complicated fractions, I believe that what is of importance is a thorough grounding in fundamental principles. As the student matures mathematically his technique should improve so that by the time it is necessary to handle complicated cases he can do so.

No one would question the importance of simple cases of factoring such as ax^2+bx+c, including perfect squares and the difference of two squares, the sum and difference of two cubes, and an expression of four terms factorable by grouping or by regarding it as the difference of two squares or as a perfect cube. Other forms such as $x^4+x^2y^2+y^4$, $a^n \pm b^n$; and expressions containing five or six terms are unnecessary. And yet I recall one text which employs the quotient of a^n-b^n by $a-b$ in obtaining the derivative of x^n for integral n. But the binomial theorem can be used instead, or, better still, the desired result may be obtained by repeated application of the fundamental rule for differentiating a product.

As I have already indicated, one should be familiar with the application of the fundamental operations to radicals, and, in addition, with the extraction of the square root of a number and with the rationalization of numerator or denominator of such functions as $\dfrac{a-\sqrt{x}}{a+\sqrt{x}}$ and $\sqrt{\dfrac{a-x}{a+x.}}$ The extraction of the square root of a binomial surd is of no importance.

As regards the theory of exponents the laws for $a^m a^n$, $a^m \div a^n$, $(a^m)^n$, a^0, a^{-m}, where a may be any function, are important. But many problems in our texts and on college entrance examinations are not of a kind the student meets later, although they may be of value in developing or showing his understanding of the laws.

Another essential is the theory of logarithms as given in our texts exclusive of the rule for change of base. The solution of exponential equations is useful in a very few problems and also in so far as it shows how to take the logarithm of both sides of an equation of the form $a^b = c$, an important process.

Turning now to the fundamental question of algebraic equations, it is unnecessary to say that the solution of one or more equations which can be reduced by the removal of fractions and radicals to linear and quadratic equations, numerical or literal, is indispensable. In addition, it is desirable that the student should be able to determine whether two linear equations in two variables have one solution, are inconsistent, or have an infinite number of solutions. And in the last case, he should be able to find as many solutions as may be necessary. The solution of a system of linear equations, homogeneous as well as non-homogeneous, the conditions for a solution, and the elimination of the variable from such a system, by means of determinants might be applied very frequently and to great advantage in analytics, and occasionally in the calculus. Very few texts, however, use determinants at all consistently. To do so would necessitate the addition of three or four theorems to the chapter in algebra. And while I have no settled opinion in this matter, I believe it would be advantageous for the Association to discuss at some future meeting whether or not a more consistent use of this invaluable mathematical tool in elementary work is desirable.

It is essential that one be able to solve a quadratic equation by completing the square, for this process is frequently used in other connections. As regards simultaneous quadratics, the case in which one equation is linear is of first importance, and the only others which are important are pairs of equations such that the elimination of one variable may be effected by obvious methods. The usual method for treating symmetrical

equations is occasionally convenient but not essential, and the so-called homogeneous quadratics, which are usually not homogeneous, are of no use.

Too great emphasis cannot be laid upon the fundamental importance of the determination of the nature of the roots of a quadratic by means of the discriminant, and the expressions for the sum and product of the roots in terms of the coefficients may be used occasionally.

Of course the work in analytics proceeds much more readily if the student has made free use of graphs. So much has been said concerning graphs in recent years that I shall pass over them with the remark that in my opinion not even our most recent texts have made as much use of graphs as is desirable for a complete understanding of elementary subjects, quite apart from the advantages analytics and calculus would derive from a more extended use of graphic methods. But the use of graphs in algebra should be carefully restricted to the clarifying of the concepts of algebra, unless, indeed, and here we come in contact with an entirely different question, it is desirable to correlate certain portions of analytics with the more elementary work. I mention this explicitly because I have heard the derivation of the equations of conic sections from geometric definitions discussed as a part of graphic algebra.

Before leaving the subject of algebra permit me to emphasize the important topics negatively. The student of analytics and calculus has occasional use for the binomial theorem, for the rule for squaring the sum of several terms, for a continued proportion, for the sign of a quadratic (best determined, as is the sign of any function, from its graph), for the limit of a fraction as a variable becomes infinite, for the meaning of variation, and for the facts that a polynomial of degree n may be written as the product of n linear factors, that imaginary roots of a polynomial with real coefficients occur in conjugate pairs, and that two roots of a polynomial become infinite when and only when the coefficients of the two highest powers of the variable approach zero. But I do not believe that his work would be seriously hampered by not having had these things in advance.

I have found use for the rule for the extraction of the real cube root of a number but once since I studied it, and on that

occasion a student then studying algebra asked me to explain it. Nor do I find any use for the extraction of square and cube roots of polynomials, except such as are obviously perfect squares or cubes. Equations in which the reciprocals of x, y, z are treated as unknowns do not arise, and one sees only very simple equations in the quadratic form. But if equations in the quadratic form are solved by setting a single variable for a function, the student becomes acquainted with one of the most fruitful methods of mathematics, namely, change of variable. The idea of ratio is fundamental but the technique of proportion is not. There is little use for arithmetic and none for harmonic progressions. The infinite geometric series is useful, but with the possible exception of this series, I believe that the whole topic of infinite series is best left to the calculus. The binomial theorem for fractional exponent, the exponential and logarithmic series are derived so simply by Taylor's formula that it seems to me a useless waste of time to derive them by algebraic methods. And unless the latter is done the mere question of convergence of an infinite series is hardly worth considering.

The texts in analytics and calculus make little use of the theory of equations except as already indicated, but they might very well do so. Permutations and combinations are of no importance, and as the first course in analytics and calculus deals only with real numbers, no intimate knowledge of complex numbers is necessary. Partial fractions are used in a fundamental problem of the integral calculus, but by the time a student needs them his algebraic technique is sufficiently developed to handle them readily. I say this with assurance because I have taught the calculus to a few classes whose preparation had not included partial fractions.

Plane trigonometry may be dismissed with the remark that all of the usual course is essential, except the formulas for the solution of oblique triangles for which there is little or no use. The importance of the solution of the right triangle and of all of the analytic side of the subject is inestimable.

Comparatively few theorems of plane geometry are actually used, but a general knowledge of the subject is essential. In terms of the syllabus of the National Committee of Fifteen, an

adequate preparation would be given by considering the theorems in black face type, in italics and Roman. Some of the latter could undoubtedly be omitted, while of those in small type I have noticed but one for which I recall a use, namely, that a circle is determined by three points. But the omission of this theorem would cause no difficulty. Even fewer theorems of solid geometry are needed. Except for the rules for the areas and volumes of prisms, pyramids, cylinders, cones and spheres (but not for truncated solids, frustums, spherical pyramids, etc.) the student needs only an idea of how things look in space, or in other words, only facts which are intuitively evident and which may be stated in terms easily understood.

One topic, algebraic in its nature but applied chiefly in geometry, namely, the theory of limits, is used fundamentally in the calculus. But in recent years there has been a decided leaning toward the belief that a formal treatment of this subject is unsuitable for high school students and that it is best postponed until later.

Thus many portions of high school mathematics are not used explicitly in the study of analytics and calculus. These subjects are taught in Yale College without being preceded by a course in advanced algebra and in Princeton and Dartmouth to students who have not had solid geometry. I do not approve of the plan at Yale. And if the Princeton-Dartmouth scheme is worth consideration the discussion must not be of the question, Can solid geometry be discarded? but rather, Can we replace solid geometry by something more valuable?

What is needed for the problems of analytics and the calculus is a considerable knowledge of geometry for their proper orientation, and for their solution, the solution of equations and the transformation of functions, both algebraic and transcendental.

In conclusion, let me say most emphatically that the development of mathematical maturity and technique sufficient for the successful pursuit of analytic geometry and the calculus requires more than a study of the facts there used. Whatever may be the topics the student pursues, he needs at least as much mathematical training as he now receives.

THE UNIVERSITY OF ROCHESTER.

WHAT IN HIGH SCHOOL MATHEMATICS IS OF MOST IMPORTANCE AS A PREPARATION FOR ANALYTICAL GEOMETRY AND THE CALCULUS IN COLLEGE?

By Maurice J. Babb.

I asked this question of one of my sophomore classes and they all answered, "Algebra!!" Then I asked "What part of algebra?" and they answered, "Simplification of all kinds of expressions, notion of + and —, transposition, substitution of other expressions for 'unknowns,' radicals, use of fractional and negative exponents, binomial theorem, solution of equations in one and two unknowns (either of first and second degree), quadratic equations, *especially the theory* (this last very emphatic), expressions containing the logarithmic notation, graphs, and the language."

Trigonometry they believed was next. The most important parts to them were definitions of the sine, cosine, etc., and their elementary relations, expression of any part of a right triangle in terms of two other parts, circular measure, graphs, a few formulas, much work in identities, accurate interpolation and the language.

They were surprised how few geometric facts they used and how comparatively seldom they used them. The square on the hypotenuse loomed up large, then the area of a triangle and especially that of the equilateral triangle, the circumference and area of a circle, loci, volumes of different solids especially solids of revolution, relations of homologous lines, areas and volumes of similar figures. Some geometry of the circle, and a few definitions such as those of a plane, and of parallel lines. This knowledge they needed in a generalized form and they admitted that it was the training in logic rather than the facts that made geometry helpful.

In arithmetic, the use of the fundamental operations with accuracy and dispatch seemed to have been lost to some extent and the decimal point bobbed up in unexpected places.

A Little Physics for Applications in Mechanics.—Everywhere I found a lack of knowledge of the "Principle of Permanence or no Exception." They, like some Romans of old, were desirous for the "new thing" rather than its interpretation in the language of the old.

Now I am rather inclined to discount the teacher who is always complaining about the preparation of his pupils. He is generally a poor teacher himself. Boys and girls forget a great many things from day to day and more from year to year; but there are poor teachers in the high schools as well as in the colleges and in the "grades." I have personal knowledge of the following incidents: An eight-grade teacher this last spring started the elements of algebra as given in Hamilton's Arithmetic. The questions were like the following:

$$\begin{array}{r} +6x \\ -4x \\ +2x \\ \hline \end{array}$$

In explaining it she told her pupils to subtract all the numbers from the first. The next day, being a frank and conscientious teacher, she told them she had made a mistake and that they should have added all the numbers. Now what could you do with that class after all that? Forty out of forty-two failed on the examination for entrance to the high school. She said it was lack of preparation and got away with it.

In my classes in pedagogy of secondary mathematics I have frequently found a considerable portion of the class who did not know anything about the theory of quadratic equations. They were entirely innocent of any relation between the sum and product of the roots of a quadratic equation and the coefficients thereof. Nor did they know of any way of finding out the character of those roots except by completing the square or use of the formula, and only then when it came out even. One teacher said the reason she liked algebra was because the answer always came out even.

In geometry many bright and experienced teachers told me a plane was shaped much like the samples Wanamaker sends out of his dress goods, and one teacher who has now turned to the ministry (a reverse process from that of some years ago),

having drawn two cones side by side on the board, stated that one of them was a right cone, and in answer to further inquiry explained that the other was a left cone. These are all classroom experiences with successful practicing teachers presumably much above the average, and I could tell you some more.

I believe that the most important thing in high school mathematics or any other subject is a more thorough and intimate acquaintance with at least the elementary parts of the subjects to be pursued in college. This should be gotten by intelligent study from well-written texts and under the direction and criticism of a well-trained and skillful teacher. If this standard cannot be obtained at once it should be approached as rapidly as possible.

Then the teacher should realize where to draw the line, where to place the high lights, where to merely emphasize by drill, where to give some or all an inkling of what it is all about. The magazines long ago understood the psychology of "continued in our next." But be it remembered that mathematics for the teacher and mathematics for the boy are very different things. The teacher wants to know all the boy might ask; the boy wants to discuss all he sees the present necessity of, and can be stimulated by occasional forecasts of the future. The teacher should be able to understand every method the boy may try. Many a budding genius is crushed by uniformity of method. There is also some food for thought in the fact that Greek mathematics developed out of the attempt to solve the three impossible problems of antiquity. How like the old struggle against fate! "There is no royal road to geometry," and the doctrine of interest does not necessarily include the doctrine of ease. Knowledge is power and there is nothing effeminate about it.

All mathematicians know that a nomenclature is chosen after careful trial, because it is perspicuous to the eye and elastic enough to suit the purpose. We generally give the boy the nomenclature first and when he wants to solve a problem he tries all the formulas he can think of. A formula is useful only when it is not needed. A symbolism is good only when it is a crying necessity and a pleasure to the user.

There is a mathematical language. An eminent English cler-

gyman named Sydney Jordan had a son who went up to matriculate. His father was much afraid of his examination in Euclid and told Jordan, Jr., to telegraph as soon as his fate was known. The young hopeful sent the following telegram: "Hymn 342, verse 5, last three lines." The father got out the hymn-book and read these words:

> Sorrow vanquished,
> Labor ended,
> Jordan passed.

It was in language he could understand. Now many a bright mind could work mathematics if he only understood the symbolism. Any sensible boy believes the truth of many propositions in geometry, but it often takes the best minds a long while to discover that the analytic part of the proof has been omitted and that the theorem has been polished by the wise men of the ages as lovingly and delicately as Clark ever polished an object glass for a telescope, until it got its present definite shape. These theorems have been proved and improved. Some of the most elementary propositions like the preface need to be written last.

Another thing that affords difficulty is that the pupil starts to work the problem before he has examined all the conditions. A Sunday-school teacher was striving to make Noah a real man to his boys. He asked them what they thought Noah did all the time in the ark. One boy suggested that he might have been fishing. The teacher said possibly he might. Another boy, who had been thinking quietly, said he didn't think he could have fished long, "'cause he only had *two* worms."

Because of the difficulty to translate into algebraic language it is almost impossible to get a result to a reading problem on a college examination paper unless it is a well-known variation to some stock problem and done by the aid of pure memory. Ordinary writing is wonderful to the fellow who doesn't know how. Mathematical language is as difficult as shorthand to the girl who doesn't have the key to the symbolism. Yet girls learn shorthand in a comparatively short time.

Many a wealthy man is unable to relish the dainties set before him and longs for the poor man's appetite. If they have the

hunger they can be fed. A few years ago I had a boy in my sophomore class of civil engineers. Six years before that time he landed in New York an ignorant and almost penniless Yiddish lad who couldn't write his name in his own language and who understood no English. He said the only way he could be a gentleman was to be a scholar. At the time he was in my class he owned enough real estate to pay his dues in the building and loan, his university fees and for his board and clothing. What we all need is a good appetite and then proper food. But I do not believe in a predigested diet for a healthy mind. A bright boy cannot thrive on the same rations as his duller classmate and will, like Oliver Twist, rebel against the board.

The French teachers of mathematics are mathematicians, and their bright pupils are three years ahead of ours at the end of the high school course, while in Germany the entire class is at least two years ahead, both in subject matter and thoroughness. As to the reasons advanced for this evident superiority abroad in England, France and Germany, I refer you to the various magazine articles on the Perry movement, Klein's work in Germany and the more recent work in France. I would advise you to use these new ideas, as I would advise the boy with his problems: study the conditions carefully before you go to work.

Now I am going to tell you about some troubles we have in analytical geometry and calculus at the University of Pennsylvania. Transpositions offer great difficulty on account of the artificial way they are taught and the fact that the right-hand side of an equation is supposed forbidden ground for the unknown. Why not

$$70 - 3x = 14 - x + 6x,$$
$$56 = 8x,$$
$$x = 7.$$

Also

$$x/24 - 5x/42 = 17/36 - 11x/56$$

gives

$$x/12 - 5x/21 = 17/18 - 11x/28.$$

Now

$$12 = 2 \times 2 \times 3,$$
$$21 = 3 \times 7,$$
$$18 = 2 \times 3 \times 3,$$
$$28 = 2 \times 2 \times 7.$$

Therefore multiply by $2 \times 2 \times 3 \times 3 \times 7$, etc.

Never multiply out till you are forced to, but keep in the factored form

$$\frac{1}{1-x}+\frac{6}{x}=\frac{6-5x}{1-x},$$

$$\frac{6}{x}=\frac{5-5x}{1-x},$$

$$\frac{6}{x}=5,$$

$$\frac{6}{5}=x.$$

Birds of a feather flock together in mathematics. Very few boys seem to know the definition of a root of an equation and would have been satisfied with all the answers that generous algebra would give.

Proportion has been sadly neglected of late.

$$\frac{1+z}{1-z}=\frac{n+1}{n},$$

$$\frac{2z}{2}=\frac{1}{2n+1}.$$

division and composition

$$z=\frac{1}{2n+1}.$$

Also solve for n.

Solve for x in the following:

(*a*) $\sqrt{x^2-x+2}=z-x,$

(*b*) $\sqrt{2+x-x^2}=(2-x)z.$

In radicals

$$\sqrt{6}\times\sqrt{10}=2\sqrt{15},$$

immediately on account of the common $\sqrt{2}$. This also is applicable to imaginaries where we take out the required number of common factors including the $\sqrt{-1}$.

Thus

$$\sqrt{-8}\times\sqrt{-6}=\sqrt{-2\times4}\times\sqrt{-2\times3}$$
$$=-2\times2\sqrt{3}=-4\sqrt{3}.$$

Every factor which has a partner can emerge, and those that have no duplicate have to stay where they were.

Very few boys examine their results in problems like the following:

$$\sqrt{x-3}-\sqrt{2x-5}=\sqrt{x-6},$$
$$\sqrt{x}\pm\sqrt{y}=\pm\sqrt{a}.$$

How difficult it is to isolate the x's and y's:

$$Ax+By+C=0,$$

$$\frac{x}{-\frac{C}{A}}+\frac{y}{-\frac{C}{B}}=1,$$

or

$$y=-\frac{A}{B}x-\frac{C}{B},$$

or from

$$b^2x^2+a^2y^2=a^2b^2$$

obtain

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

and

$$y=\pm\frac{b}{a}\sqrt{a^2-x}.$$

The simplification of

$$(x-ae)^2+y^2=e^2\left(x-\frac{a}{e}\right)^2.$$

$$e>1 \text{ and } e<1$$

offers great difficulties because the boys cannot pick out the coefficients of the different powers of x.

Of course anybody can do almost any problem if he lives long

enough, but what we want is the correct answer in a reasonable time and with simple examples that means "instanter."

$$a^2(1-e^2)=b^2,$$

$$e=\frac{\sqrt{a^2-b^2}}{a}.$$

Now construct e, a/e and ae, having given a and b.

Eliminate K from

$$x=-\frac{ma^2k}{m^2a^2+b^2},$$

$$y=-\frac{b^2k}{m^2a^2+b^2}.$$

Elimination is not always done by addition and subtraction. The function idea and the definition of a root are closely allied. We can substitute root for proof.

Evaluate

$$3x^2-6x+4y^2+4y-5=0,$$

when

$$x=1,\ 2,\ x_1+1,$$

$$y=-2,\ 2,\ y_1-\tfrac{1}{2}.$$

Factoring and finding roots are two variations of the same thing.

The factor theorem is very important, but factoring often is overdone, as only the simplest cases are really needed.

Simplify $\dfrac{x^4-2x^3+7x+14}{2x^3-4x^2+6x-12}$.

Improper fractions are tabooed in elementary calculus. Thus:

$$\frac{2x^2-3}{3x^2-7x}=\frac{2}{3}+\frac{14x-9}{3x(3x-7)}.$$

From theory of quadratics we have, if

$$\begin{cases} y^2=4ax, \\ x=sy+t, \\ y^2-4asy-4at=0, \end{cases}$$

therefore

$$\frac{y_1 + y_2}{2} = 2as.$$

and the diameter of a parabola is parallel to the axis.

Similarly if

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, \qquad y = mx + k,$$

then, when roots are equal,

$$y = mx \pm \sqrt{a^2m^2 - b^2},$$

and we have the equation of a tangent with a given direction or slope.

Eliminating y in this same system of equations we obtain

$$(a^2m^2 - b^2)x^2 + 2mka^2x + a^2(k^2 + b^2) = 0.$$

Our tangent at infinity, viz., the asymptote, is given when the two roots are infinite, that is when the coefficients of the highest powers in succession $= 0$; whence

$$m = \pm b/a, \; k = 0,$$

hence

$$y = \pm b/a \cdot x \text{ is equation of asymptote.}$$

This theory of the quadratic is in all the elementary algebras, and, like all solutions of single or simultaneous equations is much clearer when illustrated by graphs.

In trigonometry much facility is gained by having the pupil translate directly from the figure without going through the geometry as a medium. Much of the trouble is a matter of notation

$$\cos(\sin^{-1}n)$$

means if the sine of an angle is n what is its cosine. The projection of a line equals its length multiplied by the cosine of the angle between the line and its projection.

In geometry it is very important that the student be familiar with the solids and areas of revolution and their manner of generation.

Suppose an isosceles triangle inscribed in a circle of radius v. Let its altitude be y and its base $2x$, then $y(2v-y)=x^2$ from the geometry of the circle and we have either the base or altitude in terms of the other. Now try the similar problem for the circumscribed isosceles triangle, and the inscribed rectangle. Then revolve and obtain the sphere with its inscribed and circumscribed cone and inscribed cylinder and obtain the relations between the radius of the base and the altitude in terms of any given volume or surface.

Most of the processes of algebra are very artificial and uninteresting until illuminated by the calculus. Some day we may do as the French and Germans do, teach a good deal of the calculus and analytic geometry in the high school, but this will not properly come until long after our teachers will have illuminated the high school mathematics through their knowledge of its most powerful and interesting applications, the analytic geometry and the calculus. Let us hope for more power, more knowledge.

UNIVERSITY OF PENNSYLVANIA,
PHILADELPHIA, PA.

EDUCATIONAL VALUES IN MATHEMATICAL TEACHING.

By H. E. Hawkes.

Any member of the teaching profession—a profession which is closely bound by the demands of routine work—is likely to be tempted either to assume radical and extreme views, or else to become stereotyped and to follow tradition. Just which direction any individual will follow is largely a matter of temperament. But either extreme usually betokens a failure to see things in their true relations, and in the proper perspective.

It seems to me that meetings of associations like this offer admirable opportunity to look at some of the larger aspects of our professional responsibility, to attempt to place our various methods and aims in some perspective, and to determine some principles of procedure armed with which we may return to our routine of details with fresh enthusiasm and renewed confidence in the worth of our labors.

During recent years much thought and experiment has been brought to bear on the attempt to determine some norm or standard in terms of which the various subjects of study in our schools may be compared and possibly measured. One important phase of the problem, though far from being completely worked out, serves to indicate results which may be stated roughly as follows:

In the study of any subject, as for instance, mathematics, we are first met by a large number of specific results in the nature of facts acquired, theorems proved, principles mastered, which distinguish it from other subjects. This category of results I will call the *subject matter*.

These facts are acquired and elaborated by the use of certain methods involving mental processes which are quite distinct from the subject matter itself. In this second category, which we may call *mental processes*, we place memory, the power of analysis, generalization, reasoning and the like.

As a result of this working over and mastery of the subject,

qualities of a quite different character are evoked which we may call *moral qualities*, such as perseverance, patience, reliability, response to duty, appreciation of the true, the good and the beautiful.

Of course these three categories cannot be defined with precision, and very likely no two men would agree exactly as to their content, but I think it is clear that they form a convenient way of dividing into segments what may be a continuum of intellectual experience.

Now it is claimed, and indeed it seems quite reasonable to anyone on a moment's reflection, that the contents of the first category are but slightly transferable from the field in which they were acquired into other fields. For instance, the facts about the angles of triangles which are learned in geometry are not carried over spontaneously and easily even into so closely allied a field as that of physics. In a certain sense, then, the subject matter is of narrow educational value. Its significance is intensive rather than extensive.

The second category is transferred quite easily. The rational processes, for example, which the boy learns to use in his mathematics plays him in good stead when, in the law school, he is trying to follow the effects of legal principles. The relation between the educational content of the first two categories was well expressed some years ago by President Hadley of Yale when he said "the value of an education largely consists in studying facts which will not be used in after life by methods which will be used."

The third division of moral or spiritual qualities are of indefinitely wide transferability, and in this sense constitute the elements of most extensive educational value in the study of any subject.

Let me hasten to add that the acquirement of the second and the third class of results is impossible without the first. No boy can get the mental and moral value which the study of mathematics should afford merely by hearing someone talk about it. Only by steady and hard work with the details of the subject will the richer benefits be derived. Excepting for the man who will occasionally use mathematics in his professional career, the subject matter in and of itself is of little importance. It is a

(*To be continued.*)

NEW BOOKS.

Monographs on Topics of Modern Mathematics. Edited by J. W. A. YOUNG. New York: Longmans, Green & Company. Pp. 424. $

The object of this book as stated by the editor is "to bring within reach of secondary teachers (in service or in training), college students, and others at a like stage of mathematical advancement, a scientific treatment of some of the regions of advanced mathematics that have points of contact with the elementary field. Undoubtedly one of the most crying needs of our secondary instruction in mathematics to-day is that the scientific attainments of the teacher be enlarged and their mathematical horizon widened; and I believe that there is a large body of earnest teachers and students that are eager to extend their mathematical knowledge if the path can be made plain and feasible for them."

The titles of the monographs are as follows: The fundamentals of geometry; Modern pure geometry; Non-Euclidean geometry; The fundamental propositions of algebra; The algebraic equation; The function concept and the fundamental notions of the calculus; The theory of numbers; Constructions with ruler and compasses; The history and transcendence of π.

They seem to be carefully written and well suited to meet their object.

Brief Course in Analytic Geometry. By J. H. TANNER and JOSEPH ALLEN. New York: American Book Company. Pp. 316. $1.50.

This book is an abridgment of the Elementary Course in Analytic Geometry by the same authors and preserves the important features of that book. It includes besides plane, an introduction to solid analytic geometry and a chapter on higher plane curves. It would seem to contain an abundance of well-selected material in both theory and examples for a first course.

The Teachers' Practical Philosophy. By GEORGE TRUMBULL LADD. New York: Funk and Wagnalls Company. Pp. 336. $1.25 net.

The purpose of this book is "to emphasize the personal and moral elements as those which, broadly understood, must be relied upon to secure the needed improvements in our schools to produce men and women of the right sort to conduct safely, wisely and righteously the affairs of church and state. The author believes that the lack of discipline through moral and religious motives and in accordance with moral and religious ideals, in the home-life, in school, and in society at large, is the prime source of all our national evils so far as they are connected with the educative process as now in vogue." It contains interesting

chapters on: The functions of the teacher; The equipment of the teacher; The chief ideals of the teacher; The teacher's relation to society and the state.

Complete Business Arithmetic. By GEORGE H. VAN TUYL. New York: American Book Company. Pp. 432. $1.00.

In the preparation of this book the following specific objects have been kept in view: A training that leads to facility and accuracy in handling the fundamental operations; the placing of emphasis on the fundamental principles of arithmetic rather than upon set rules for the solution of problems; clearness and fullness of explanation; and the providing of problems that have an informational value. The chapter on aliquot parts, as applied to billing, trade discount, and simple interest, is placed early in the text. Common and decimal fractions are treated together, as is the case in business. A great many problems are provided for mental work. Many of the problems are taken from the business affairs of corporations, cities, states, and nations of the world.

Inheritance of Acquired Characters. By EUGENIO RIGNANO. Translated into English by BASIL C. H. HARVEY. Chicago: The Open Court Publishing Co. Pp. 413. $3.00.

This work appeared first in French in 1906 and later in German and Italian. The author, who was a student of physics as well as of biology, attacks the problem from the physical side and offers in this book an explanation on a physical basis, of assimilation, cell division, and the biogenetic law of recapitulation in ontogeny, and suggests a mechanism whereby the inheritance of acquired characters may be effected. He says: "Some deny flatly the possibility of ever arriving at an understanding of the nature of life. But if we ask ourselves in what this understanding of the nature of life could consist, from the point of view of positive philosophy, we have no difficulty in recognizing that such an understanding must be reduced to comparing vital phenomena with some physico-chemical model already known, suitably modified by the particular special conditions imposed upon it so that just these special conditions shall determine the differences which exist between the vital phenomenon and that phenomenon of the inorganic world most closely related to it. If this be so, it is then the duty of science emphatically to refuse to give up the attempt to understand the nature of living matter, for that would be to belie the spirit of all scientific endeavor. For whether it be clearly recognized or not, it is just this search for the nature of the vital principle which properly constitutes the principal object and final goal of all biologic study in general."

Mental Discipline and Educational Values. By W. H. HECK. New York: John Lane Company. Pp. 208.

The main purpose of this book is to modify the doctrine of formal discipline and upon such a modification to establish a standard of educa-

tional values. A second purpose has been to sum up and organize the discussions and experiments in regard to the disciplinary value of studies.

It gives a good account of the discussion of formal discipline that has been going on for some years and though few would to-day accept the extreme view of the formalists it would seem that the author leaned rather far in the opposite direction. Too much confidence in the doctrine of nervous localization will lead in this direction.

Teachers will find it interesting reading and a valuable summary of the discussion.

On Life After Death. By GUSTAV THEODORE FECHNER. Translated into English by HUGO WERNEKKE. A new edition revised and enlarged. Chicago: The Open Court Publishing Company. Pp. 134. 75 cents net.

Fechner, a professor of physics, was also interested in psychology and by combining the two became one of the founders of the science of "Psychophysics," based upon the interrelation between sensation and nervous activity. He believed in the immortality of the soul, but not in just the way most people are accustomed to think of it. He holds that the spirits of the dead continue to exist as individuals in the living, and though the reader may find the ideas expressed in an unusual way, they will certainly stimulate thought.

Memories and Studies. By WILLIAM JAMES. New York: Longmans, Green & Company. Pp. 411. $1.75 net.

Professor James's son has done a good work in collecting a number of his father's popular addresses and essays in book form. Though everything found here has already appeared in print in magazines or otherwise, they were more or less inaccessible to the general reader. Professor James is always interesting and seems particularly so in the topics of this volume which are somewhat varied. The book is in splendid type and presents a very pleasing appearance. Teachers will find it not only interesting but instructive.

The Teaching of High School Mathematics. By GEORGE W. EVANS. Boston: Houghton, Mifflin Company. Pp. 94. 35 cents.

This volume is one of the Riverside Educational Monographs edited by Henry Suzzallo and treats in concise form of the following topics: The Modern Point of View; The Order of Topics; Equations and Their Use; Some Rules of Thumb; Geometry as Algebraic Material; The Graphical Method; The Bases of Proof in Geometry; The Method of Limits; Simpson's Rule and the Curve of Sections; The Teacher.

The Teaching of Geometry. By DAVID EUGENE SMITH. New York: Ginn and Company. Pp. 339. $1.25.

An interesting book, giving Dr. Smith's viewpoint on the methods and subject matter of geometry. The book first discusses the reasons for the

study of geometry, and the discussion is sane and liberal. Following this is a brief history of the subject, touching only on points of some importance. The rest of the book is taken up with a discussion of methods, and a rather full discussion of the subject matter, definitions, axioms, and propositions. The part on the subject matter is quite closely in accordance with recent committee reports, notably with that of the National Geometry Committee of Fifteen of which Dr. Smith is a member.

In some of the discussions of methods there is a touch of dogmatism, for, on many of the mooted questions, one may express an opinion, but can hardly claim the right to decide with certainty, since the experience of others may give equal force to arguments on the other side. A rather amusing logical oversight occurs when the author condemns the book that does not contain full proofs, on the ground that, as the proofs could be obtained from other texts, such books encourage dishonesty, and then later, in speaking of methods with a full text, says "It is usually a good plan to give the easier propositions as exercises before they are reached in the text, where this can be done," although the proofs are evidently in the same book.

Although Dr. Smith qualifies his use of design by such expressions as "Teachers who feel it necessary or advisable to go outside the regular work of geometry for the purpose of increasing the pupil's interest or of training his hand in the drawing of figures . . .," his book shows to some extent the influence of the movement toward concrete settings for geometric exercises. That he does not intend this to be overdone is clear to one reading the book carefully, but might not be clear to the casual reader.

Diary and Time Saver for 1912. Chicago: Laird and Lee. 25 cents.

A very useful and inexpensive diary, better than some that cost twice as much.

NOTES AND NEWS.

MEMBERS: Will you not take the trouble to interest other teachers of mathematics who are not members of our association to join at once? Those who join and pay $1.00 now will receive five numbers of THE MATHEMATICS TEACHER and receive a receipt for their dues for the year.

THE reports of several of the American committees of the International Commission on the Teaching of Mathematics are now ready for distribution, and may be had gratis by addressing the Bureau of Education, Washington, D. C.

These reports have been carefully prepared and should be read by every teacher of mathematics. Not all of them will be equally interesting to every teacher, as some of the reports deal with questions relating to colleges and technical schools while others deal with questions relating to elementary and secondary schools.

THE Philadelphia Section held a meeting October 8, at which Miss Stackhouse read a paper on "the Principle of No Exception," and Professor E. Doolittle gave an illustrated talk on stellar astronomy.

THE Rochester Section held a meeting November 4. There were four papers: (1) "Berkeley's Mysticism of Mathematics," by C. W. Watkeys, of University of Rochester; (2) "Suggestions from Psychology for Teachers of Mathematics," by W. H. Sherk, Lafayette High School, Buffalo; (3) "Equivalence of Equations," by W. Betz, East High School; (4) "Correlation of Freshman Mathematics at the University of Rochester," by A. S. Gale.

THE N. Y. Section made the seventeenth meeting of the Association its autumn meeting. The next meeting will discuss phases of arithmetic teaching.

THE Syracuse Section will hold its meeting December 29, at the High School, Syracuse, N. Y.

ANYONE present at the recent seventeenth meeting or any one reading in THE TEACHER the papers by W. A. Ballou and H. E. Hawkes, cannot but feel how futile it must be to try to realize the point of view of each of the speakers in one single course in mathematics. We will have to have pure mathematics and applied mathematics as separate courses in secondary school as well as in college.

THE SEVENTEENTH MEETING.

THE annual meeting was called to order by the President, Dr. William H. Metzler, at 10:45 A. M., December 2, in the chapel of Teachers College. After the reading of the minutes the preliminary report of the committee on testing the results of arithmetic teaching was given by Dr. Rorer, the chairman. The report was received and the committee was continued. The preliminary report for the committee on geometry teaching was given by Mr. E. R. Smith. The report was received and the committee was continued. The committee appointed at the previous council meeting to consider the question of better affiliations among the eastern associations of mathematics teachers was continued and asked to report at the next meeting. It was also voted that it would be better for the Association if the sections should provide for their own finances rather than avail themselves of the promise in the by-laws which gives to each section the right to twenty-five per cent. of the dues it pays in. The nominating committee was appointed as follows: Mr. E. R. Smith, chairman, Dr. Hallett, Mr. D. D. Feldman.

The first paper of the meeting was given by Mr. W. A. Ballou, of Pratt Institute, on "Alligation applied to the Calculations of Analytical and Industrial Chemistry." The morning session was concluded by a paper by Mr. Maurice J. Babb, of the University of Pennsylvania, on "What in High School Mathematics is of most Importance as a Preparation for Analytical Geometry and Calculus in College?"

At the afternoon session the auditing committee, consisting of Philip R. Dean, chairman, and Clifford B. Upton reported that

the Treasurer's accounts were correct and the report was read and accepted.

Receipts.

Cash, November 26, 1910	$226.67
Dues to December 1, 1911	393.05

Expenditures.

New York Section	$ 25.00
Philadelphia Section	18.50
Rochester Section	5.50
Syracuse Section	5.00
Secretary	1.00
Treasurer	30.33
General expenses	93.49
Algebra Syllabus Committee	16.70
MATHEMATICS TEACHER	250.00
	$445.52
Balance, December 1, 1911	$174.20

The following officers were elected: *President:* Isaac J. Schwatt, University of Pennsylvania, Philadelphia, Pa.; *Vice-President:* Albert E. King, Erasmus Hall High School, Brooklyn, N. Y.; *Secretary:* Howard F. Hart, High School, Montclair, N. J.; *Treasurer:* Susan C. Lodge, Philadelphia Collegiate Institute, Philadelphia Pa.; *Members of Council:* Arthur S. Gale, University of Rochester, Rochester, N. Y.; Jonathan T. Rorer, William Penn High School, Philadelphia, Pa. For unexpired term of I. J. Schwatt, Harry English, director of Mathematics of High Schools, Washington, D. C.

The second topic of the morning was continued in a paper by A. S. Gale, University of Rochester. Then Herbert E. Hawkes, of Columbia University, gave a paper on "Educational Values in Mathematical Instruction." The last paper of the session, by the Secretary, considered the "Provisional Report of the National Committee of Fifteen on a Geometry Syllabus." An interesting discussion followed, opened by Mr. E. R. Smith. The discussion led to the adoption of three motions: (1) That the list of standard theorems should not be shortened for the purpose of affording time for problems in concrete setting; (2) that the incommensurable case be not required; (3) that the committee be asked to prepare a maximum list of theorems

NEW MEMBERS.

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ST. NICHOLAS IN 1912.

THE true story of "Whittier and the Little Quaker Girl," and the meeting between them, with many charming pictures, will be one of the good things in *St. Nicholas* during 1912.

A sketch of unusual interest in an early number of *St. Nicholas* is promised in "The School of Four Thousand Welcomers," an account of a marvelous dramatic performance designed and carried out by the pupils of one of our public schools.

India's rich store of folk-lore has been drawn upon for a series of jungle-life tales by Ellen Babbitt, and illustrated in silhouette by Ellsworth Young. "Jataka Tales" will be published in *St. Nicholas* during 1912.

A single brief paper in a recent number of *St. Nicholas*, entitled "Locating a Claim," made so deep an impression and was so widely copied that its author, George Lawrence Parker, was promptly commissioned to prepare a series of similar talks for the new volume of *St. Nicholas*. They will discuss helpfully and inspiringly such topics as "Care and Don't Care," "On Being Somebody," "Reading the Signals," "Yes and No," "The World We Live In," and "Making Good and Making Better."

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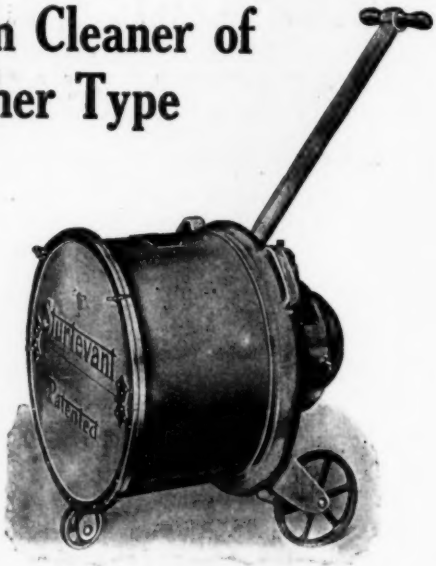

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